Individual Evolutionary Learning
in the market environment with full or limited information*

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Abstract

In this paper we explore how specific aspects of mechanism design affect the efficiency of the market outcome. In particular, we are interested whether the availability of information about actions of other participants improves market efficiency. We consider a simple market of a homogeneous good populated by buyers and sellers. The valuations of the buyers and the costs of the sellers are given exogenously. Agents are involved in the consequent trading sessions, which are organized either as a call auction or as a continuous double auction with electronic book. Using Individual Evolutionary Learning mechanism agents submit price bids and offers, trying to learn the most profitable strategy by looking at their realized and counterfactual or “forgone” payoffs. In this setting we compare the outcomes of the call auction and the continuous double auction under the open and closed book treatments, focusing both on the informational and allocative efficiency of markets and also on the individual learning outcome.

Keywords: Learning, Call Auction, Continuous Double Auction, Market design

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1 Introduction

A question of “What makes markets allocatively efficient?” has attracted a lot of attention in the recent years. Influential research on Zero-Intelligence (ZI) traders (see Gode and Sunder (1993) and Gode and Sunder (1997) for which the above question makes a title) have suggested that the rules of the market (and not individual rationality) may be mostly responsible for allocative efficiency. For example, continuous double auction (CDA) market is allocatively efficient even when the subjects behave randomly, subject to budget constraints.

The debate on the allocative efficiency is not over, however. Initial contribution of Gode and Sunder lead to substantial critical response (see Duffy (2006) for a review). Recently Gjerstad and Shachat (2007) argued that assumption made by Gode and Sunder that the ZI traders are “constrained”, i.e., cannot trade with negative profit, should be attributed to the individual rationality and not to the market structure. LiCalzi and Pellizzari (2008) have shown that the allocative efficiency of the CDA would drop substantially if the order book would not clear after every transaction, as Gode and Sunder assume in their simulations. The allocative efficiency would be restored, however, if agents would learn between periods, for instance using sophisticated Gjerstad and Dickhaut (1998) algorithm. For alternative market design, call auction (CA) Satterthwaite and Williams (2002) demonstrate that strategic consideration by traders can lead to inefficiency, if their number is finite.

This discussion leads to a question of the role played by individual rationality in the market. Actual participants are definitely not Zero-Intelligent, and it is not obvious that efficiency achieved under ZI would carry over the CDA market with participants, who are learning or behave strategically. On the other hand, experiments in Kagel, Harstad, and Levin (1987) and Lei, Noussair, and Plott (2001) provide evidence that participants can occasionally violate the individual rationality requirement and trade with clear losses. The role of learning in the market environment is, therefore, interesting to analyze.

When the behavior of agents on the model reminiscent the behavior of actual participants, one can address some further question on the role of market design. For instance, one may wonder whether information available to participants play some role in allocative efficiency.

In this paper we analyze the effect of information available at the market on its allocative and also informational efficiency. Participants are modeled as neither entirely rational (theoretical question of how fully rational participant should behave in the CDA is still open), nor irrational. Instead, they learn their strategies (submitted orders) participating in a similar market repeatedly. The learning algorithm which we employ, the Individual Evolutionary Learning (IEL) was introduced in Arifovic and Ledyard (2003). It can be considered as a simplified version of the genetic algorithms developed to better suit the need of economists. Two main Darwinian ideas are inherent to this mechanism. First is mutation, which means that agents are allowed to use, in principle, any strat-
egy at some period of time. In particular, the requirement of individual rationality is not imposed, so that agents are allowed to trade with losses. Second is selection with reinforcement, so that strategies with higher past payoffs have higher representation in the strategy pool and higher probability to be used. Furthermore, agents are comparing strategies not only on the basis of actual but also of forgone payoffs.

Since the CDA environment is rather complicated for an analysis, we consider also the CA under which agents are submitting their orders to the central authority, “market-maker” who clears the market. Similarly, Arifovic and Ledyard (2007) study the IEL in the CA, but we focus more on the learning outcome.

The rest of the paper is organized as follows. In the next section we explain all the details of our model. Section 3.1 is devoted to the analysis of learning in the CA. Here in particular, we are interested in whether individuals are able to learn to submit their individual valuations or cost. In Section 3.2 we simulate the market operating under CDA protocol.

2 Model

We start with describing environment and defining the competitive equilibrium as a benchmark against which we shall compare our simulations. We then proceed by explaining two types of market architectures for which the model is simulated. We discuss, in particular, the informational difference between the open and closed book market settings. Finally, we explain the Individual Evolutionary Learning mechanism.

Environment

Suppose we have a fixed number $B + S$ market participants, $B$ buyers and $S$ sellers. Sellers each owns one unit of commodity and buyers each want to consume one unit of the good. Every round $t$ these agents trade in the market with each other. Throughout the paper index $b \in \{1, \ldots, B\}$ denotes the buyer and the index $s \in \{1, \ldots, S\}$ denotes the seller.

At period $t$ every buyer’s valuation good is given by $V_{b,t}$, which he receives if he buys a unit. Every seller produces a unit of commodity at cost $C_{s,t}$. The valuations and costs are bounded within the interval $[0, \eta]$.

It is assumed that each buyer knows his valuation $V_{b,t}$ and each seller knows his cost $C_{s,t}$. The traders do not know the valuations and costs of others. A buyer who trades in period $t$ at price $p$ derives utility $V_{b,t} - p$, while a seller who trades derives utility $p - C_{s,t}$. No trade brings zero utility. To summarize, the trader’s utility as a function of
transaction price is given as

\[
U_{b,t}(p) = \begin{cases} 
V_{b,t} - p & \text{if buyer } b \text{ traded at price } p \\
0 & \text{otherwise}, 
\end{cases}
\]

\[
U_{s,t}(p) = \begin{cases} 
p - C_{s,t} & \text{if seller } s \text{ traded at price } p \\
0 & \text{otherwise}. 
\end{cases}
\]

(2.1)

We will distinguish between two types of environment. A situation in which the valuations of every buyer and cost of every seller are fixed over time is called a fixed environment. This situation is common to the theoretical, simulation and experimental literature, for corresponding examples see Satterthwaite and Williams (2002), Arifovic and Ledyard (2007) and Gode and Sunder (1993). In the fixed environment \( V_{b,t} = V_b \) and \( C_{s,t} = C_s \). Alternatively, we will also deal with general situation of changing environment, when the valuations and costs of every trader can change between trading sessions. The reason for our interest in the changing environment is two-fold. First, we are interested in whether agents learn some strategy of order submission with respect to their current values/costs (e.g., whether they submit their current values/costs, which makes sense in the CA). Comparison of fixed with changing environment may show whether such learning is simpler in one of them. Second, many real markets represent changing environment, and the analysis of learning in such environment is interesting and important question per se. For instance, in financial market agents will change their valuations with new information about assets, as their expectations are changing.

Given the set of reservation values and costs, \( \{V_b\}_{b=1}^B \) and \( \{C_s\}_{s=1}^S \), one can derive the step-wise aggregate demand and supply curves, whose intersection determines the competitive equilibrium. This outcome will serve as a theoretical benchmark, as it maximizes the mutual benefits from trade. More specifically, the intersection of demand and supply determines the unique\footnote{This is guaranteed by assuming that in a special case when there exist a buyer whose reservation value coincides with the cost of a seller, these sellers and buyers trade maximum possible quantity.} equilibrium quantity \( q^* \geq 0 \) and, in general, an interval of the equilibrium prices \( [p^*_L, p^*_H] \). See Fig. 1 for illustration of one of our configurations. For any price within the interval (shown by the red lines), only inframarginal units trade in the competitive equilibrium (those units which are to the left from the equilibrium quantity). The extramarginal units do not trade (units to the right from the equilibrium point). Those buyers and sellers who trade at some equilibrium price derive non-negative utility, while those who do not trade get zero utility. The sum of all utilities of buyers and traders (which do not depend on the equilibrium price) give the allocative value of the trade, which is maximized when the transaction price belongs to the equilibrium interval.
2.1 Call Auction

We model the call auction (CA) as a so-called sealed-bid auction, as in Arifovic and Ledyard (2007). Namely, we assume that at period $t$ every buyer submits one bid, $b_{b,t}$, and every seller submits one offer (ask), $a_{s,t}$. No information is revealed during the process of order submission. When all bids and asks are submitted, the market is “called”. An equilibrium quantity $Q_t$ and equilibrium price $P_t$ are computed in a standard way as intersection of the step-wise demand and supply function constructed on the basis of all orders. In case if there are many clearing prices, the clearing price $P_t$ is set to the middle of the interval of the clearing prices. In other words, the market operates as $1/2$—double auction.

Formally, let us rank the submitted bids and asks as $b_1 \geq b_2 \geq \cdots \geq b_B$ and $a_1 \leq a_2 \leq \cdots \leq a_S$. Let $k$ be the highest number such that $b_k \geq a_k$ ($k = 4$ in example of Fig. 1). Set $Z = \min\{b^k, a^{k+1}\}$ if $k < N$, and $Z = b^k$, otherwise. Analogously, set $z = \max\{a^k, b^{k+1}\}$ if $k < N$ and $z = a^k$, otherwise. Then any price within the interval $[z, Z]$ will clear the market. We choose $P_t = (z + Z)/2$ as the cleaning price for period $t$.

Every buyer $b$ whose bid $b_{b,t}$ is above than or equal to $P_t$ buys a unit of the good and earns a payoff of $V_{b,t} - P_t$. Every seller $s$ whose ask $a_{s,t}$ is below than or equal to $P_t$ sells a unit of good and earns a payoff of $P_t - C_{s,t}$. These trading agents will be called inframarginal traders for this trading round, while all other traders who do not trade and receive the payoff of 0 are extramarginal traders. To summarize, the actual payoffs...
in the CM are defined as

\[
U_{b,t}^{CM} = \begin{cases} 
V_{b,t} - P_t & \text{if buyer } b \text{ is inframarginal} \\
0 & \text{otherwise},
\end{cases}
\]

\[
U_{s,t}^{CM} = \begin{cases} 
P_t - C_{s,t} & \text{if seller } s \text{ is inframarginal} \\
0 & \text{otherwise}.
\end{cases}
\]

Note that in the CA there exists a unique price, and therefore the payoffs of the traders do not depend on their bids directly. The only thing which matters for individual payoff is whether a trader is inframarginal or extramarginal.

Two CA designs which we simulate differ only in a type of feedback which traders receive between trading sessions. In the call auction-open book (CA-OP) design each agent is given full information about all bids, offers and prices from the previous period (trader does not know, however, the identity of bidders and hence do not have a direct access to the behavioral strategies used by others). In the call auction-closed book (CA-CL) design, the agents are informed only about the price from the previous round (they, of course, also know their own own bid).

Let \( \bar{C}(t) \) denote the set of all buyers’ bids and sellers’ asks submitted at time \( t \), i.e.,

\[
\bar{C}(t) = \big\{ b_{b,t}, a_{s,t} \mid b \in \{1, \ldots, B\}, s \in \{1, \ldots, S\} \big\}.
\]

Under the closed book design this whole set is not known to buyers and sellers: they know only their own bids and asks as well as equilibrium price. Thus, under the CM-CL design the information sets of buyers and sellers are given as

\[
\mathcal{J}_{b,t}^{CM-CL} = \{b_{b,t-1}, P_t\} \cup \mathcal{J}_{b,t}^{CM-CL}, \quad \mathcal{J}_{s,t}^{CM-CL} = \{a_{s,t-1}, P_t\} \cup \mathcal{J}_{s,t}^{CM-CL}.
\]

Let

\[
\mathcal{E}_{-b}(t) = \bar{C}(t) \setminus \{b_{b,t}\}
\]

denote the set of all submitted orders, except for the order of buyer \( b \). Under the open book market the information set of buyer \( b \) can be written as

\[
\mathcal{J}_{b,t}^{CM-OP} = \mathcal{E}_{-b}(t) \cup \{b_{b,t}\} \cup \mathcal{J}_{b,t-1}^{CM-OP},
\]

so that it enhances not only on the buyer’s own bid but also on the set of all bids and asks. This leads to possibility of the following counterfactual analysis that buyer \( b \) will perform during learning stage. Note that when the book is open, every buyer knows \( \mathcal{E}_{-b}(t) \) and for arbitrary own bid \( b' \) can compute notional equilibrium price \( \bar{P}_t(b') \). Now buyer can simply find his foregone utility which he would extract at period \( t \) if instead of submitting bid \( b_{b,t} \) another bid \( b' \) would be submitted. Analogously, the seller \( s \)’s
information set

\[ J_{s,t}^{CM-OP} = C_{-s}(t) \cup \{a_{s,t}\} \cup J_{s,t-1}^{CM-OP}, \]

can be used to infer notional equilibrium price \( \tilde{P}_t(a') \) which would clear the market given that all traders submit orders as in \( C_{-s}(t) = \mathcal{C}(t) \setminus \{a_{s,t}\} \) and seller \( s \) submits arbitrary order \( s' \).

2.2 Continuous Double Auction

The market organized as a continuous double auction (CDA) reminds the electronic trade common to the stock exchanges nowadays. The main difference from the CA is that the trade during one period \( t \) is not simultaneous but sequential. When submitting an order, every trader, in principle, have an access to some information about the orders submitted before him during this session. However, in order to facilitate the comparison with the CA set-up, in our simulations we assume that the traders do not use this information.

Thus all the orders, bids \( b_{t,s} \) of the buyers and asks \( a_{s,t} \) of the sellers, are generated before the trade starts on the basis of commonly available information set. Every trader visits the market once during a trading session, and the sequence in which the traders are arriving to the market to submit their orders is determined randomly every trading session. The market clearing is accommodated by an electronic order book which stores all unsatisfied orders. Given the traders’ orders and sequence of traders’ arrival to the market, this mechanism operates as following. If the newly submitted order finds a “matching order,” it is satisfied by the price of this matching order. A matching order is defined as an order stored in the opposite side of the book at whose price the transaction with the new order is possible. If there are many orders which match the incoming order in this sense, the matching order is one of them, chosen according the price-time priority.\(^2\) If the submitted order does not find a matching order, it is stored in the book. After every trader arrived at the market, i.e., at the end of the trading period, the book is cleared by removing all the unsatisfied orders. Next trading session starts with the empty book.

In such a market there is no unique price during period \( t \). Instead, over the trading session the price is changing and we can identify the transaction price with those traders whose transaction has been recorded at this price. The buyer \( b \) who made a transaction at the price \( p_{b,t} \) gets a payoff of \( V_{b,t} - p_{b,t} \), while the buyer who did not trade over the session gets 0. The seller \( s \) who succeeded in selling the unit at price \( p_{s,t} \) gets payoff of \( p_{s,t} - C_{s,t} \), while the seller who did not trade gets 0. Thus, the actual payoffs in the CDA

\(^2\)For instance, if the buy order with bid \( b \) is arriving, the sell side of the book is checked. If there are sell orders whose prices less or equal than \( b \) in the book, the transaction takes place at the minimum price among such orders. In case if there are multiple minimum prices, the unit of the seller submitted his order earlier is sold.
are given by

\[
U_{b,t}^{\text{CDA}} = \begin{cases} 
V_{b,t} - p_{b,t} & \text{if order } b_{b,t} \text{ of buyer } b \text{ is satisfied} \\
0 & \text{otherwise}
\end{cases}
\]

\[
U_{s,t}^{\text{CDA}} = \begin{cases} 
p_{s,t} - C_{s,t} & \text{if order } a_{s,t} \text{ of seller } s \text{ is satisfied} \\
0 & \text{otherwise}
\end{cases}
\]

(2.4)

Note that in the CDA market the payoff of the trader strongly depends on the submitted order (but also on the sequence of trades). Indeed, for transaction between traders \( b \) and \( s \), the price of this transaction, denoted by our convention as \( p_{b,t} \) and \( p_{s,t} \), coincides with either \( b_{b,t} \) or \( a_{s,t} \).

Analogously to the CA, two CDA market designs, differed in a type of information feedback, will be considered. In the continuous double auction-open book (CDA-OP) design each agent is given full order book of the previous session. Thus, an agent have information about all bids and asks from the previous period, as well, as about the order in which these bids and asks arrived. With this information a trader can reconstruct the book and find the prices of all transactions. As before, trader does not know the identity of bidders. In the continuous double auction-closed book (CDA-CL) design, the agents, apart from own bid and payoff, are informed only about the average transaction price, \( P_{t}^{av} \), observed during the previous trading session.\(^3\)

The information sets for CDA-CL and CDA-OP designs can be formally written analogously to the case of CA. Recall that \( \mathcal{E}_{-}(t) \) denote the set of all bids and asks submitted at time \( t \), except for the bid of buyer \( b \). Let \( \mathcal{O}(t) \) contain the sequence in which the orders arrived to the market. Then under the open book design, the information set of a buyer is

\[
\mathcal{I}_{b,t}^{\text{CDA-OP}} = \mathcal{E}_{-}(t) \cup \mathcal{O}(t) \cup \{ b_{b,t} \} \cup \mathcal{I}_{b,t-1}^{\text{CM-OP}},
\]

Since this buyer knows the order of arrival of all offers, he can make a counterfactual analysis to compute own payoff for any alternative bid \( b' \). Analogously the information set for seller is defined. Instead, under closed book design, the information set of a buyer is

\[
\mathcal{I}_{b,t}^{\text{CDA-CL}} = \{ b_{b,t}, P_{t}^{av} \} \cup \mathcal{I}_{b,t-1}^{\text{CM-OP}},
\]

2.3 Agent Behavior

In this paper we will ignore all strategical aspects of the traders behavior. We are interested, instead, in the outcome of the market under simple evolutionary learning mechanism, which reinforces successful and discourage unsuccessful strategies. As we will see during their learning process agents can indeed explore that they may have certain

\(^3\)Alternatively, we can provide agents with only closed price of the session. The average price is, however, more informative, especially in the market with few traders.
power in the market, which can be used to their strategical advantage.

Independent of the market clearing mechanism, an observed action of every agent during a trading round is one submitted order. But how do these orders generated? The generation of the orders is modelled by the Individual Evolutionary Learning (IEL) algorithm, which requires to specify a class (or space) of alternative strategies (or messages) mapping information to the actions, limit for every trader this space to the small time-varying individual-specific pool of strategies, choose one message from the pool using probabilistic approach, and, finally, change the pool to the next period on the basis of performances.

**Messages**

Let us assume for simplicity that all possible orders belong to the continuous interval \([0, \eta]\). We assume that messages represent the deviation of intended submitted order from the trader’s valuation/cost. Messages are explicitly conditioned on the trader’s valuations/costs. For buyer \(b\) the space of possible messages is given by a set \([V_{b,t} - \eta, V_{b,t}]\). A given message \(\varepsilon_{b,t} \in [V_{b,t} - \eta, V_{b,t}]\) translates to a submission of the bid

\[
b_{b,t} = V_{b,t} - \varepsilon_{b,t}.
\]

Analogously, for seller \(s\) the space of messages is defined by a set \([-C_{s,t}, \eta - C_{s,t}]\) and a message \(\varepsilon_{s,t} \in [-C_{s,t}, \eta - C_{s,t}]\) translates to the submission of the sell order

\[
a_{s,t} = C_{s,t} + \varepsilon_{s,t}.
\]

Note that our assumption that the orders fall in the time-invariant interval \([0, \eta]\) is satisfied. However, in general, traders are permitted to submit ask orders above their reservation values and sell orders below their costs. This situation corresponds to negative messages \(\varepsilon_{b,t}\) and \(\varepsilon_{s,t}\). We can distinguish two modeling options. We say that in presence of *individual rationality* (IR) negative messages are forbidden. This situation was considered in Arifovic and Ledyard (2007). Opposite situation, in which negative messages are permitted, occurs under *no individual rationality constraints*. In such a case we let our traders to evolve and learn not to submit orders which lead to individual losses.

**Individual Pool**

Even if there is a continuum of possible messages, every agent will be restricted at every time to choose between a limited amount of them. The pool of messages available for submission at time \(t\) by buyer \(b\) is denoted by \(B_{b,t}\). The pool of messages available for submission at time \(t\) by seller \(s\) is denoted by \(A_{s,t}\). Every period the pools of agents are slightly changing but the number of messages in the pool is fixed and equal to \(J\), which is one of the key parameters of our model. Some of the messages in the pool might
be identical, so that in reality every agent chooses every time from $J$ or less possible alternatives.

The pool used at time $t$ will be updated between two trading periods by subsequent application of two algorithms, experimentation and replication. During experimentation stage, any message from an old pool can be substituted with a small probability by some new message, which is randomly generated in a neighborhood of the old message. In such a way for every buyer and seller the intermediate pools $B_{b,t+1}'$ and $A_{s,t+1}'$ are formed. Experimentation thus represents a local and relatively rare mutation of old pool. At the replication stage two randomly chosen messages of just-formed pool are compared one with another, and the best of them left in the pool. The comparison is made according to the relative performance of these messages during past periods. Such process is repeated $J$ times independently for every agent, so that the new pools $B_{b,t+1}$ and $A_{s,t+1}$ are formed. During replication we, therefore, increase an amount of “successful” messages in the pool at the expense of not so successful messages.

When the new pool is formed, one of the messages is drawn randomly with certain selection probability and corresponding order is submitted for trading session $t + 1$. An important feature of this learning mechanism is that experimentation, replication and selection probability are all based upon foregone utilities from the previous period.

### Calculating the Foregone Utilities

The actual payoffs can always be computed according to (2.2) for the CA and (2.4) for the continuous double auction. However, learning agent will try to infer his foregone utility from other alternative strategy. In this analysis agent is necessarily boundedly rational, since he ignores the analogous learning process of all the other agents. Actually, every agent is involved into counterfactual analysis, given that other agents do not change their behavior. The calculation is made according to (2.1), where the price of transaction depends on the type of market and amount of information which is available to the agent.

Let us start with the CA. When the book is open, i.e., under the CM-OP design, the agent knows all the orders submitted by the other agents. For example consider a buyer. We denoted the set of orders at time $t$ without buyer $b$’s order as $\mathcal{C}_{-b}(t)$. Taking these $B + S - 1$ orders as given, a buyer constructs the aggregate demand/supply schedules for every own message $\varepsilon_{b,t}$. He then computes (notional) competitive price in this market which would be observed had he submitted a different order using another message $\tilde{P}_{t-1}(V_{b,t-1} - \varepsilon_{b})$. The payoffs extracted at this price are foregone utilities of
this agent. They are computed using (2.2)

\[ U_{b,t}(\varepsilon_b|^{CM-OP}_{b,t-1}) = \begin{cases} V_{b,t} - \tilde{P}_{t-1}(V_{b,t-1} - \varepsilon_b) & \text{if } V_{b,t-1} - \varepsilon_b \geq \tilde{P}_{t-1}(V_{b,t-1} - \varepsilon_b) \\ 0 & \text{otherwise} \end{cases} \]

\[ U_{s,t}(\varepsilon_s|^{CM-OP}_{s,t-1}) = \begin{cases} \tilde{P}_{t-1}(C_{s,t-1} + \varepsilon_s) - C_{s,t-1} & \text{if } C_{s,t-1} + \varepsilon_s \leq \tilde{P}_{t-1}(C_{s,t-1} + \varepsilon_s) \\ 0 & \text{otherwise} \end{cases} \]

When the book is closed, i.e., under the CM-CL design the agent knows only equilibrium price \( P_{t-1} \). To compute the foregone payoff agent assumes (perhaps wrongly) that he would always have an opportunity to trade at price \( P_{t-1} \). Thus, his foregone payoffs are

\[ U_{b,t}(\varepsilon_b|^{CM-CL}_{b,t-1}) = \begin{cases} V_{b,t} - P_{t-1} & \text{if } V_{b,t-1} - \varepsilon_b \geq P_{t-1} \\ 0 & \text{otherwise} \end{cases} \]

\[ U_{s,t}(\varepsilon_s|^{CM-CL}_{s,t-1}) = \begin{cases} P_{t-1} - C_{s,t-1} & \text{if } C_{s,t-1} + \varepsilon_s \leq P_{t-1} \\ 0 & \text{otherwise} \end{cases} \]

Consider now the CDA market. Under the open book design, CDA-OP, agent knows order in which all traders visited the market and orders which they submitted. Consequently, for any alternative message, agent \( b \) can reconstruct a whole book, find the (notional) price of his transaction, \( p^*_b,t \), and find his own payoff using (2.4). Thus, the foregone utilities of an agent are given by

\[ U_{b,t}(\varepsilon_b|^{CDA-OP}_{b,t-1}) = \begin{cases} V_{b,t-1} - p^*_b,t(\varepsilon_b,t) & \text{if order } V_{b,t-1} - \varepsilon_b \text{ of buyer } b \text{ is satisfied} \\ 0 & \text{otherwise} \end{cases} \]

\[ U_{s,t}(\varepsilon_s|^{CDA-OP}_{s,t-1}) = \begin{cases} p^*_s,t(\varepsilon_s) - C_{s,t-1} & \text{if order } C_{s,t-1} + \varepsilon_s \text{ of seller } s \text{ is satisfied} \\ 0 & \text{otherwise} \end{cases} \]

Under the closed book design the order book cannot be reconstructed and, hence, agent can use only average price, \( P^\text{avg}_{t-1} \) of the previous session as an indication for possible realized price given alternative message submitted. Analogously to the CA with closed book, agents foregone utilities are

\[ U_{b,t}(\varepsilon_b|^{CDA-CL}_{b,t-1}) = \begin{cases} V_{b,t} - P^\text{avg}_{t-1} & \text{if } V_{b,t-1} - \varepsilon_b \geq P^\text{avg}_{t-1} \\ 0 & \text{otherwise} \end{cases} \]

\[ U_{s,t}(\varepsilon_s|^{CDA-CL}_{s,t-1}) = \begin{cases} P^\text{avg}_{t-1} - C_{s,t-1} & \text{if } C_{s,t-1} + \varepsilon_s \leq P^\text{avg}_{t-1} \\ 0 & \text{otherwise} \end{cases} \]
Individual Evolutionary Learning

For a given buyer \( b \) the pool at time \( t - 1 \) is given by \( B_{b,t-1} \), and for a given seller \( s \) the pool at time \( t - 1 \) is a subset of the set \( A_{s,t-1} \). The pool contains \( J \) messages, generated initially at random. After every trading session the following learning process takes place for every agent. We present the process for a buyer \( b \).

First, the old pool of messages, \( B_{b,t-1} \), is updated through the *experimentation*. Each message is deleted from the pool with small probability of experimentation, \( \rho \), or stay in the new pool, \( B'_{b,t} \) with probability \( 1 - \rho \). In case, if a message was deleted, the new message, close to the deleted one, is included in the pool. We generate the new message as follows. Let us say that the message \( \varepsilon_{b,t-1} \) has been deleted. We generate a new message according to

\[
\varepsilon_{b,t} := \varepsilon_{b,t-1} + \xi,
\]

when there are no IR constraints, and as

\[
\varepsilon_{b,t} := |\varepsilon_{b,t-1} + \xi|,
\]

when there are IR constraints. Note that in Arifovic and Ledyard (2007) there was

\[
\varepsilon_{b,t} := \max\{0, \varepsilon_{b,t-1} + \xi\}.
\]

In these formulas \( \xi \sim \mathcal{P}(0, \sigma^2_e) \), where \( \mathcal{P} \) is a distribution with mean 0 and variance \( \sigma^2_e \). Thus, two parameters, \( \rho \) and \( \sigma^2_e \) are crucial for experimentation. First of them measures a probability that experimentation with a given element from the pool takes place, while the second measures the “closeness” of the modified message to the original one as a result of experimentation. We simulate the model with two distributions \( \mathcal{P} \), which are uniform and normal. In addition we should also check that the implied order \( V_{b,t} - \varepsilon_{b,t} \) falls into interval \([0, \eta]\). We enforce this constraint by repeating the draw of \( \xi \) until the constraint is satisfied.

Second, for each strategy in the new pool \( B'_b \) the hypothetical utility is computed. Now *replication* takes place and the final new pool, \( B_{b,t} \), is formed. During the replication, every place in the new pool is filled by the winner of the tournament selection between two strategies from the old pool, these two strategies being randomly chosen with uniform probability and with replacement.

Finally, given new pool \( B_{b,t} \) and information set \( I_{t-1} \) (which varies depending on the type of market) the selection probability of each particular message \( \varepsilon_{b,t} \) from the new pool is computed as

\[
\pi_{b,t}(\varepsilon_{b,t}) = \frac{U_{b,t}(\varepsilon_{b,t}|I_{t-1}) + \eta}{\sum_{\varepsilon \in B_{b,t}} (U_{b,t}(\varepsilon|I_{t-1}) + \eta)}.
\]
3 Simulations and Results

The Table 1 summarizes the parameters of our model and present the values which we used in the benchmark simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value (Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval of valuation/costs</td>
<td>([0, \eta])</td>
<td>([0, 1.2])</td>
</tr>
<tr>
<td>Number of strategies in a pool</td>
<td>(J)</td>
<td>100</td>
</tr>
<tr>
<td>Number of buyers and sellers</td>
<td>(B = S)</td>
<td>5</td>
</tr>
<tr>
<td>Probability of experimentation</td>
<td>(\rho)</td>
<td>0.03</td>
</tr>
<tr>
<td>Distribution of experimentation</td>
<td>(P(0, \sigma^2))</td>
<td>(N(0, 0.01^2))</td>
</tr>
<tr>
<td>Individual Rationality constraint</td>
<td>IR</td>
<td>enforced</td>
</tr>
</tbody>
</table>

Table 1: Parameter values used in simulations.

In addition to basic parametrization we tried various designs for open book and closed book auctions by combining two distributions for experimentation, normal and uniform, \(P = N, U\), different probabilities of mutation, low and high, \(\rho = 0.03, 0.3\), different standard deviations of experimentation, low and high, \(\sigma = 0.01, 0.1\) and two scenarios when Individual Rationality (IR) constants are (1) enforces and (2) relaxed.

3.1 Call Auction

Figure 2: Summary of the aggregate outcomes for the CA under closed (left panel) vs. open (right panel) book. IR constraints enforced, normal distribution in experimentation with parameters \((\rho, \sigma) = (0.03, 0.01)\). Red lines indicate equilibrium price range, equilibrium efficiency and equilibrium number of transactions.

Figure 2 shows market aggregates, such as price, efficiency and number of transactions for the CA. Under the closed book we observe generally more price fluctuations, than under the open book. Both auctions achieve similar level of efficiency and number of transaction. For deeper inside, in Figure 3 we show the evolution of the individual trading strategies (bids/asks) over time.

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Figure 3: Individual bids (left) and asks (right) in the CA under closed (top) vs. open (bottom) book. IR constraints enforced, normal distribution in experimentation with parameters $(\rho, \sigma) = (0.03, 0.01)$. Red lines indicate equilibrium price range, equilibrium efficiency and equilibrium number of transactions. Stars denote valuations/costs of agents, vertical line - equilibrium price range.

Under the closed book marginal traders (who determine price) can not realize that their orders influence the price and na"ively think that previous price will hold. They get the same level of forgone payoff for any bid/ask between their valuation/costs and the previous price and therefore submit random orders in this region. This, in turn, causes price fluctuations. Under the open book trades understand consequences of changing their bids/asks and submit bids/asks strategically. This strategic behavior leads to the coordination on some price which may change only because of experimentation and randomness. Moreover, in the open book we observe competition to be a marginal trader whenever own costs/valuations permit (in case traders are inframarginal). Extramarginal traders (who do not trade) in both cases submit their bids purely randomly. Boundedly rational strategic behavior, that is the desire to be a marginal trader assuming that others will not change their orders and high level of randomness impairs efficiency and may lead to lower number of transactions under the open book.
Increasing probability of experimentation and standard deviation of noise leads to higher fluctuations. Impact is more profound under the open book. Using uniform distribution in experimentation also increases fluctuations compared to the case when normal distribution is used.

Individual learning: under the open book inframarginal agents coordinate on equilibrium price, under the closed book marginal agents coordinate on price range between marginal buyer’s valuation and marginal seller’s costs.

When we remove individual rationality constraints (agents are allowed to have negative foregone payoff), we observe more price volatility. The influence on price volatility of closed auction is more profound. The effect of higher probability of experimentation and higher standard deviation are amplified when no individual rationality constraints are imposed.

Individual learning: under the closed book, marginal trades learn not to submit orders resulting in negative utility, under the open book traders still coordinate on price.

3.2 CDA

Figure 4: Summary of the aggregate outcomes for the CDA under closed (left panel) vs. open (right panel) book. IR constraints enforced, normal distribution in experimentation with parameters \((\rho, \sigma) = (0.03, 0.01)\). Red lines indicate equilibrium price range, equilibrium efficiency and equilibrium number of transactions.

Figure 4 shows market aggregates, such as price, efficiency and number of transactions for the CDA, while Figure 5 shows the evolution of the individual trading strategies (bids/asks) over time.

Similarly to the CA under the CDA we observe more price volatility when the book is closed. The efficiency is generally higher, however, under the closed book. Overall the volatility of price is higher and efficiency is lower under the CDA than under the CA. Additional noise is added by random arrival of trades to the market.

Under the closed book CDA traders tend to learn their valuations/costs. This happens under CDA, but not under the CA, because price spans larger area and the gap
of positive foregone payoff between the agents’ valuations/costs and the price is small at some periods. Higher probability of experimentation and higher standard deviation increase deviations from agents’ valuations. Under the open book CDA agents who trade coordinate on one price in attempt to "squeeze" the margin. In the CDA each agent, who trades, determines the price, while in the CA only marginal trader determines the price.

Higher level of experimentation leads to faster convergence to one price while higher standard deviation introduces more price distortions. The observations under the uniform noise are fairly similar.

The removal of individual rationality constraints slightly impairs learning of agents’ valuations/costs under the closed book and the coordination to one price under the open book. Under the open book CDA this results in some periods of relatively low efficiency and low number of transactions.
References


