Venture Capitalists’ Portfolio Choices

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DRAFT

Abstract

In the presence of scarce resources, the ability of a venture capitalist to add value to his portfolio through screening and monitoring decreases as the portfolio size increases in the number projects included. In this setting, a theoretical model of portfolio composition of venture capital firm indicates that through their compensation scheme, the general partners have an incentive to shift their portfolio composition away from small projects in favor of large projects (i.e. projects that require a larger investment) as the amount of funds provided by limited partners increases. The affect of the portfolio decision on project flow through the multi-stage financing process is examined.

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Introduction

The venture capitalist is a vehicle able to allocate investment funds into new project ideas that otherwise would have difficulty raising funds from other financial institutions.\footnote{I use the term venture capitalist to identify limited partners in a venture capital firm.} This is typically due to the absence of collateralizable assets in the business plan of the project. The accommodating debt structure is thus to provide the venture capitalists partial ownership of the new business. The venture capitalist also provides non monetary resources in the form of monitoring of the project’s development and business advice and contacts. These services contribute to develop the project from an idea to a revenue generating business. The typical funding model involves financing in stages in which only enough investment capital to develop the project to the next stage. At each stage, the project is reevaluated for continued investment. An important element of the stage financing model is that the initial first-stage investing is done by a single originating venture capitalist while subsequent financing involves a syndicate. These features are documented in, for example, Sahlman (1990) and Kaplan and Stromberg (2003).

In the present paper, we present a model showing that, as the amount of funds available to venture capitalists expands, venture capitalists may have an incentive to shift from early stage projects, which require small investments, to late stage projects, which typically require larger investments. The essential ingredients of the model are the typical compensation of venture capitalists combined with decreasing return to scale with respect to the number of projects included in the portfolio. The analysis continues with an investigation of the impact this shift on portfolio composition, when it occurs sector wide, can have on the flow of projects through the multi-stage investment process.

The literature provides a number of explanations for stage financing and syndication as mechanisms that alleviate asymmetric information problems. Stage financing can help maintain the effort of the entrepreneur. It also allows the venture capitalist to learn about the prospect of the project’s final success during development, giving the venture capitalists the option to abandon a project revealed to offer a low expected payoff. Syndication reduces moral hazard created from the stage financing. It also allows for the distribution of risk and accommodates the need for larger scale financing that typically is required for later stage funding. See, for example, Pindyck (1993), Admati and Pfleiderer (1994), and Fluck, et al (2006).

These basic features of multi-stage financing with syndication are incorporated into a model of the flow of projects from an unfunded idea through to the venture capitalist’s divestiture of the business. The portfolio decision of the venture capitalist will affect flows and thus, in a market setting, the returns gained at each financing stage.

Venture capitalists (VCs) typically receive an annual management fee of about 2% of the capital.
committed by limited partners plus a performance fee of about 20% of the realized profits (see Litvak 2009 for an excellent recent analysis of the compensation of VCs, as well as). 

... 

Consider, now, the issue of decreasing return to scale with respect to the number of projects managed by the VC. Recent papers, for example Kanniainen and Keuschnigg (2003, 2004), have discussed the critical role of scarce resources for VC and its impact on fund size. Fulghieri and Sevilir (2009) discuss how scarce resources play a role in determining the degree of focus of a VC’s portfolio. These works assume that the amount of monetary wealth available to the VC has no impact on his portfolio choices. 

... 

There has been an ongoing debate about the VC industry having become too big (Austin (2009), Kedrosky (2009))², and VCs progressively investing in larger late-stage projects (moving away from start-ups) as money has poured into the industry, resulting in the so called equity gap.

The intuition behind our result is simple. Investing in larger projects allows the entrepreneur to manage a larger amount of money while keeping the number of projects small.

I The model

A Fund allocation

Let \( i = S, L \) denote the type of project. Projects \( S \) need \( I_S = 1 \) unit of capital while projects \( L \) need \( I_L = k > 1 \) units of capital. Let \( R_i \) denote the return (revenues) of project \( i \) in case the project succeeds and assume that project \( i \) succeeds with probability \( \rho \) (the probability of success does not depend on the type of project). If a project fails, the return is zero. Thus, the expected profits from project \( i \) read:

\[
E(\pi_i) = \rho R_i - I_i,
\]

and the expected rate of return from project \( i \) reads:

\[
E(r_i) = \frac{\rho R_i}{I_i} - 1.
\]

The VC has to decide the number of projects \( n \) to include in the portfolio and the composition \( \theta \in [0, 1] \) of the portfolio, with \( \theta \) denoting the fraction of \( L \) projects (i.e. \( \theta n \) is the number of \( L \) projects). Assume that the probability of success of a project, \( \rho \), is decreasing in the number of projects in the VC’s portfolio.

²See Kaplan and Lerner (2010) for a different view.
That is, let $\rho(n) : [0, \pi] \rightarrow [0, 1]$, $\rho(0) = 1$, $\rho(\pi) = 0$, $\rho'(n) < 0$ and $\rho''(n) \leq 0$.\(^3\)

Consider a portfolio $(n, \theta) \in A = [0, \pi] \times [0, 1]$. The amount of capital under the management of the VC which is associated to this portfolio is equal to $(1 - \theta)n + \theta nk$. Let $\Pi(n, \theta)$ denote the profits generated by this portfolio when investment is liquidated. The VC’s compensation can then be written as follows:

$$
\Phi(n, \theta) = f((1 - \theta)n + \theta nk) + c \max (0, \Pi(n, \theta))
$$

(1)

Assuming that projects are independent, the expected compensation of the VC is given by:

$$
\Phi^e(n, \theta) = f((1 - \theta)n + \theta nk) + c \rho(n) \Pi_S(1) + \theta n \Pi_L(k).
$$

(2)

Consider the specific case in which $R_S - 1 = R_L - k = \pi$. The profits expected from a project - in case the project is successful - are the same for $S$ and $L$ projects. This clearly implies that $E(\pi_S) = E(\pi_L)$, and that $E(r_S) > E(r_L)$. The expected profits of $S$ and $L$ projects are the same, while $S$ projects guarantee a higher rate of return since $I_S < I_L$. Thus, $S$ projects are superior to $L$ projects. Using this assumption, we can finally write the expected compensation of the VC as follows

$$
\Phi^e(n, \theta) = f((1 - \theta)n + \theta nk) + c \rho(n) n \pi.
$$

(3)

Assuming risk neutrality, the VC’s problem reads:

$$
\max_{(n, \theta) \in A} \Phi^e(n, \theta)
$$

s.t.

$$
(1 - \theta)n + \theta nk \leq W.
$$

(5)

Constraint (5) says that the monetary size of the fund, $(1 - \theta)n + \theta nk$, must be smaller than or equal to the maximum amount of funds $W$ that the VC is able to raise from the limited partners.\(^4\),\(^5\)

\[^3\] $\rho''(n) \leq 0$ is not a necessary condition to derive the results we will present, but makes the problem more tractable and eases exposition.

\[^4\] It is important to stress the amount of money managed by a VC is determined by the amount of funds that limited partners are willing to provide to the VC. The VC can nevertheless leave some money on the table if he deems that the total amount of funds offered by limited partners is excessive. So, in principle the VC cannot increase $W$ at his will. However, given a certain amount of funds offered by the limited partners, he can decide how much to take in.

\[^5\] The results we derive hold even in the case in which a participation constraint for the limited partners is added to the problem in the form of a minimal (after-fee) expected rate of return that the VC has to guarantee (in expectation) to the limited partners. The intuition why the results do not change is that the participation constraint indeed translates into another threshold in terms of the maximum number of projects that the VC can include in his portfolio (that is because the expected rate of return from the portfolio is strictly decreasing in $n$). Thus, with the inclusion of a participation constraint, the maximization problem would read as problem (4), with $n \in [0, \overline{\pi}]$, where $\overline{\pi} = \min(n^+, \pi)$ and $n^+$ represents the maximum number of projects that makes the participation constraint hold with equality. Thus, adding such a constraint makes the analysis a bit more complicated and exposition much heavier, without adding any new results. Clearly, this plausible as our interest here is to analyze how the portfolio choice $(n, \theta)$ is affected by changes in $W$. If we want also to analyze how $(n, \theta)$ is affected by the minimum return required by the limited partners, then the participation constraint should be included in the problem. This
The following demonstrates that since the probability of success of each project is decreasing in the total number of projects managed, a VC shifts from portfolios of small projects to portfolios large projects as $W$ gets larger.

Since $A$ is a compact set, problem (4) always has a solution (possibly at the boundaries of $A$). In fact, the solution to the unconstrained problem is

$$\theta_n^* = 1$$

$$n_u^* = \arg \max_{0 \leq n \leq \pi} \Phi^e(n, 0).$$

This result immediately suggests that when the amount of funds $W$ that the VC is able to raise in a given period is large enough, the VC prefers a portfolio of large projects to a portfolio of small projects. In addition, $n_u^*$ will be equal to $\pi$ or to an interior value $0 < n < \pi$ depending on the shape of $\Phi^e(n, 0)$. Notice that the previous result also implies that the VC will leave on the table any amount of wealth in excess of $kn_u^*$, that is $W - kn_u^*$.

Consider, now, the solution to the constrained problem, which will give a better idea of how the choice of the VC about the composition of his portfolio is affected by $W$. The Lagrangian associated to problem (4) is:

$$L = fn [(1 - \theta) + \theta k] + \rho(n) n \pi - \lambda[(1 - \theta)n + \theta nk - W]$$

The necessary (and sufficient)\(^6\) conditions for an interior the solution are:

$$\frac{\partial L}{\partial n} = 0 : fn [(1 - \theta) + \theta k] + [\rho'(n) n \pi + \rho(n) \pi] - \lambda[(1 - \theta)n + \theta nk - W] = 0$$

$$\frac{\partial L}{\partial \theta} = 0 : fn (k - 1) - \lambda n (k - 1) = 0$$

$$\lambda \geq 0, (1 - \theta)n + \theta nk - W \leq 0 \text{ and } \lambda[(1 - \theta)n + \theta nk - W] = 0$$

From (7) is obtained $\lambda = f > 0$. Therefore, the management fee $f$ represents the marginal increase in the VC’s compensation when the wealth constraint is relaxed. Put differently, when wealth is binding, for any extra dollar under management, the VC’s compensation increases by a fraction $f$ of a dollar. This implies that as the wealth constraint is relaxed, the "performance" component of the VC’s compensation $c\rho(n)n\pi$ remains constant. As I will show, this is indeed the result of the VC re-balancing the number of projects and the composition of his portfolio in favor of large projects. Indeed, substituting $\lambda = f$ into becomes even more important if we assume that the might exist a relationship between $W$ (the initial amount of fund available in a specific period) and the level of interest rates and investment returns available in that period.

\(^6\)Sufficiency is guaranteed by the assumption that $\rho''(n) \leq 0$, which guarantees that the hessian associated to the constrained problem is negative semi-definite. This is not a necessary assumption to derive our results.
(6) and (8), produces:

\[ c\pi'(n) + \rho(n) = 0 \quad \text{(9)} \]

\[ (1 - \theta)n + \theta nk - W = 0. \quad \text{(10)} \]

If an interior solution exists, it is such that the optimal number of projects does not depend on wealth, while the composition of the portfolio changes with wealth. Consider the LHS of equation (9). At \( n = 0, c\pi'(n) + \rho(n) = 1 \). At \( n = \pi, c\pi'(n) + \rho(n) = c\pi'(\pi)\pi < 0 \). Thus, there always exists a value of \( n \) such that \( c\pi'(n) + \rho(n) = 0 \). So, let \( n_c \) be the solution to equation (9). Then, the equation equation (10) determines the optimal composition, i.e. \( \theta_c = \frac{W - n_c}{\pi(n_c - 1)} \). Notice that \((n_c, c)\) is a solution to problem (4) only for \( n_c \leq W \leq kn_c \), a condition that guarantees that \( 0 \leq (1 - \theta_c) \leq 1 \). Notice that when \( W = n_c, \theta_c = 0 \) (the VC invests only in small projects). When \( W = kn_c, \theta_c = 1 \) (the VC invests only in large project. Finally, \( \frac{\partial \Phi}{\partial W} = \frac{1}{n_c(n_c - 1)} > 0 \), suggesting that as \( W \) increases from \( n_c \) to \( kn_c \), the fraction of small projects in the VC’s portfolio decreases.

It is quite straightforward (though tedious) to show that when wealth \( W \notin [n_c, kn_c] \), we have the three following situations. For \( 0 < W < n_c \), the VC invests only in small projects, choosing \((n, \theta) = (W, 0)\). For \( kn_c < W \leq kn^*_u \), the VC invests only in large projects, choosing \((n, \theta) = (\frac{W}{\pi}, 1)\) (notice that at \( W = kn^*_u \) the unconstraint solution \((n^*_u, 1)\) can be implemented). For \( W > kn^*_u \) the VC leaves the amount of funds \( W - kn^*_u \) on the table and set up a fund of size \( kn^*_u \), with \( n^*_u \) large projects.

Summing up, the solution to problem (4) reads,

\[ (n^*, \theta^*) = \begin{cases} 
(W, 0) & \text{for } 0 < W < n_c \\
(n_c, \theta_c) & \text{for } n_c \leq W \leq kn_c \\
(\frac{W}{\pi}, 1) & \text{for } kn_c < W \leq kn^*_u \\
(n^*_u, 1) & \text{for } W > kn^*_u. 
\end{cases} \]

Figure 1 shows the value function \( \Phi^*(n^*, \theta^*) = \sup_{0 < W \leq kn^*_u} (\Phi^* (W, 1), \Phi^* (n_c, \theta_c), \Phi^* (\frac{W}{\pi}, 1)) \) in the case in which \( \rho(n) = 1 - \frac{n}{\pi}, a \geq 1 \). Thus \( \pi = a \) and \( A = [0, 1] \times [a, 1] \). In this case, the expected compensation reads:

\[ \Phi^*(n, \theta) = \frac{cn(a - n)r}{a} + fn(1 - \theta + k\theta) \]

\^In fact, if we let \( W > 0, (n_c, \theta_c) \) can be thought as the solution to problem (4) when only the wealth constraint (5) is binding, and \( n \) and \( \theta \) are not bound to take values in \( A \). It follows that the value of the expected compensation associated to any pair \((n, \theta) \in A\) can never be larger than the value of expected compensation at \((n_c, \theta_c)\). That is, \( \Phi^* (n_c, \theta_c) \geq \Phi^* (n, \theta) \), for any \((n, \theta) \in A\).
The solution of the unconstrained problem is

\[
\begin{align*}
\theta_u^* &= 1 \\
n_u^* &= \frac{a(fk + cr)}{2cr}.
\end{align*}
\]

The solution that satisfies conditions (6) to (8) is:

\[
\begin{align*}
\theta_c &= \frac{2W - a}{a(k - 1)}, \\
n_c &= \frac{a}{2}, \\
\lambda_c &= f.
\end{align*}
\]

Notice that \((n_c, \theta_c) \in A\) iff \(\frac{a}{2} < W < \frac{ak}{2}\). Therefore, the function \(\Phi^e(n_c, \theta_c)\) representing how the value of the expected compensation changes with wealth when the VC optimally chooses \((n_c, \theta_c)\) is defined only over \(\frac{a}{2} < W < \frac{ak}{2}\).

Figure 1 shows \(\Phi^e(W, 1)\) (green), \(\Phi^e(W, 0)\) (blue) and \(\Phi^e(n_c, \theta_c)\) (red). When the available amount of funds is below \(\frac{a}{2}\) the VC maximizes his expected compensation by structuring a portfolio of small projects (\(\Phi^e(W, 0) > \Phi^e(W, 1)\)), when available funds are between \(\frac{a}{2} < W < W_0\), the VC chooses a mixed composition of large and small projects, which the fraction of small projects being equal to \((1 - \theta)c\), strictly decreasing as \(W\) increases from \(\frac{a}{2}\) to \(\frac{ak}{2}\), the fraction of small projects in the VC’s portfolio decreases as well. When funds available are above \(\frac{ak}{2}\), the VC invests only in large projects. Finally when funds exceed \(kn_u^*\), the VC leaves on the table the amount of money \(W - kn_u^*\) and set up a fund of size \(W = kn_u^*\), all invested in large projects.

### B Project flow

Begin analysis with a simple model of project flows. A successful project succeeds in attracting funding in two sequential investment stages. In this simple version of the model, all projects are of homogeneous quality. Selection for investment at each stage is according to a matching process. A project that is matched to a funding source receives the investment necessary to advance to the next stage of development. Adopt the notation that superscript "I" will refer to stage-1 values and "II" to stage-2 values. To invest in a stage-1 project is to invest in a "small" project from the previous analysis while investing in a stage-2 project represent investment in a "large" project. At any point in time, \(V^I_t\) is the volume of project ideas seeking stage-1 investment funds. Likewise, \(V^{II}_t\) is the volume of projects that have completed the stage-1 development process are are seeking stage-2 investment. Let \(S^I_t\) and \(S^{II}_t\) represent the rate at which projects are selected for funding from the respective pool of projects. For a representative VC model, \(S^I_t = (1 - \theta)n\) and \(S^{II}_t = \theta n\). It then becomes true that in aggregate, \(\theta = \frac{S^{II}_t}{S^I_t + S^{II}_t}\).
Figure 1: Maximum value of the compensation as a function of wealth and portfolio composition. Green line: $\Phi^c(W, 0)$; Blue line: $\Phi^c(W/k, 1)$; Red line: $\Phi^c(n, \theta)$. 

For convenience, the analysis proceeds employing continuous time and the ability to allocate funds between projects along a continuum. The volume of projects at each stage evolve over time according to the process,

$$\frac{\partial V^k_t}{\partial t} = \alpha^k_t - \beta^k_t V^k_t - S^k_t, k = I, II.$$  \hspace{1cm} (11)

The first term is the arrival rate for new business ideas looking for financing. The last term is the exit of projects from the pool through matching. The rate of exit due to simple expiration is determined by the second term.\(^8\)

A project that is matched, and thereby funded, takes a period of $h$ before it arrives at the next stage. The stage-1 investor invests $I$ to advance the project to stage-2. At stage-2, the project is sold to a new investor at price $p_t$. The stage-2 investor also has to pay $kI$ to advance the project to completion (stage-3).

The volumes in stage-1 and stag-2 pools are linked since $\alpha^I_{t+h} = S^I_t$. Analysis reveals a steady state (dropping the time subscripts),

$$V^k_{fp} = \frac{S^k - \alpha^k}{\beta^k}$$

\(^8\)An idea can be though of as expiring according to a probability increasing with the time spent in the pool without a match. This term is mean to capture the rate of exit from such a process.
so that, in particular,

\[ V_{fp}^{II} = \frac{S_{II} - S_{I}}{\beta_{II}}. \]  

(12)

At stage-3, the venture capitalist divests from the project, receiving a payment \( F_t \) from subsequent owners.\(^{9}\) Allow, for now, that \( F_t \) is a constant, \( F_t = F \).

Let \( \phi \) indicate the probability that a project advances from stage-2 to stage-3 before expiration. Its value is

\[ \phi = \frac{S_{II}}{S_{I}} = \frac{\theta}{(1 - \theta)}. \]  

(13)

Of course, \( S_{II} > S_{I} \) cannot produce a steady state stage-2 volume, so that \( S_{I} > 0 \) and \( 0 < \theta \leq \frac{1}{2} \), ensures \( \phi \in [0, 1] \).

Incentive compatibility for the risk neutral venture capitalists to fund projects with which they are matched requires

\[ \text{IC1 : } f + \phi p - 1 \geq 0, \]  

(14)

\[ \text{IC2 : } f k + F - (p + k) \geq 0, \]  

(15)

\[ \text{IC3 : } f + fk + F - (1 + k) \geq 0. \]  

(16)

The first two IC constraints insure the VCs invest in the project to which they are matched in stage-1 and stage-2 respectively. The third constraint ensures that completed projects are profitable over the two stages. A more stringent participation constraint of the limited partners which imposes profitability on the project,

\[ \text{IC4 : } F - (1 + k) \geq 0. \]  

(17)

At the steady state, with no exogenous source of noise, the only uncertainty for a stage-1 funded project is in whether it will receive stage-2 funding before it expires.

In the current environment, the representative VC’s expected compensations is now expressed as\(^{10}\)

\[ \Phi^x(n, \theta) = f((1 - \theta)n + \theta nk) + c((\phi p - 1)(1 - \theta)n + (F - (k + p))\theta n), \]

which can be more conveniently expressed as

\[ \Phi^x(n, \theta) = n(X_1(1 - \theta) + X_2\theta), \]  

(18)

where \( X_1 = f + c(\phi p - 1) \) and \( X_2 = fk + c(F - (k + p)) \). With the VC choosing between stage-1 and

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\(^{9}\) Normally, venture capitalists exit when the mature project is sold either privately or by IPO.

\(^{10}\) To maintain parsimony in the model, the VC’s compensation function does not truncate the performance reward at zero. A number of institutional and behavioral reasons can support treating the VC as though he optimizes the employed compensation function, including the VC’s own investment in the fund, non-monetary losses to reputation of a realized negative profit, or the expectation that negative profits are in the distant tail of the distribution of the fund.
stage-2 by selecting \( \theta \), the optimization remains as expressed in (4) and (5). While \( \phi \) is affected by the aggregate relative investment between stage-1 and stage-2 projects, the representative VC takes \( \phi \) and \( p \) as given. As a result, the Lagrangian is linear in \( \theta \) and thus \( \theta \) is indeterminate.

The necessary conditions for an interior solution in \( \theta \) are

\[
\frac{\partial L}{\partial n} = 0 : (X_1(1 - \theta) + X_2\theta) + \lambda((1 - \theta) + \theta k) = 0 \quad (19)
\]

\[
\frac{\partial L}{\partial \theta} = 0 : n(X_2 - X_1) + \lambda n(k - 1) = 0 \quad (20)
\]

\[
\lambda \geq 0, \quad (1 - \theta)n + \theta nk - W \leq 0 \quad \text{and} \quad \lambda[(1 - \theta)n + \theta nk - W] = 0. \quad (21)
\]

If IC3 is satisfied, then the budget constraint in (21) holds with equality so that \( n((1 - \theta) + \theta k) = W \), indicating the VC makes use of all of the funds available. The two conditions in (19) and (20) reduce to

\[
X_2 = X_1k. \quad (22)
\]

When this condition holds, the VC is indifferent between investing in stage-1 and stage-2 projects after allowing for the opportunity cost of funding each type of project. If the condition in (22) did not hold, the VC would allocate all available funds to the stage offering the greater reward.

The necessary equality in (22) can be used to solve for the price that ensures \( X_2 = X_1k \) holds,

\[
p^* = \frac{F}{1 + k\phi}. \quad (23)
\]

In aggregate, \( \phi \) does depend on \( S^I \) and \( S^{II} \) in accordance with (13). Let

\[
\phi^* = \frac{f - 1}{F + k(f - 1)}
\]

The two IC constraints in (14) and (15) place constraints on \( \phi \), and thus on \( \theta \). Both constraints imply the same lower bound on \( \phi \), \( \phi \geq \phi^* \). Figure 2 plots both \( p^* \) (upper curve) and \( \phi p^* \) (lower curve) over the valid range of \( \phi \). Note that \( p^* \) is falling as demand for stage-2 projects increases. Keep in mind that these are steady state conditions and that as \( \theta \) increases, so does \( \phi \). While the price is declining in \( \theta \) increases, the proportion of projects matched with a stage-2 investor increases producing a net increase in the expected payoff to the stage-1 investors. The price decrease is necessary to maintain balance in the portfolio allocation decision as the expected payoff to stage-1 investing increases.

Translating back into the representative VC’s decision regarding \( \theta \), the constraints on the value of \( \phi \) translate into \( \theta \in \left[ \frac{\phi^*}{(\phi^* + 1)}, \frac{1}{2} \right] \). For any value of \( \theta \) chosen within this range, there is a price that will ensure the VC is receives equal marginal benefit for investing in stage-1 and stage-2 projects. Not all choices of \( \theta \) generate the same aggregate payoff, though. The maximum aggregate payoff is achieved.
with $\theta = \frac{1}{2}$. At this rate of investment, $S^I = S^{II}$ so there is no waste of projects expiring after stage-1 development while searching for a stage-2 investor. This will change with the introduction of projects of heterogeneous quality (also if introduce project failures during development with different rates for I and II). Thus, if there were a governing authority or some other mechanism to induce cooperation on the preferred outcome, a population of independent VC could coordinate to ensure $\phi = 1$, but there is no market mechanism by which to ensure this outcome.\footnote{One can think of the price being set by a Walrasian auctioneer, in which case the price can be set to accommodate any valid realization of $\phi$.}

Consider, now, an environment in which projects can fail to reach the next stage. Let $\rho$ be the probability that a project is successfully developed from its current stage to the next stage. As in Section A, allow that $\rho$ is a decreasing function of $n$, reflecting the resource demand and constraint on the VC to manage a project through to the next stage. The VC's expected compensation can still be expressed as in (18), but with $X_1 = f + c(\rho(n)\phi p - 1)$ and $X_2 = fk + c(\rho(n)F - (k + p))$. As with the fund allocation problem in Section A, there is a solution to the unconstrained allocation decision. Since the objective function is linear in $\theta$, $\partial \Phi / \partial \theta = 0$ alone determines the globally optimal $n^u_\star$.

Distinct from the Section A analysis, the transfer price between the stage-1 investor and stage-2 investor determines the allocation with $\theta$ either at one of the corners or indeterminate. There is a unique positive price that makes the VC indifferent between the two stages, but in this setting, it is independent of $\phi$. The constraint on $\phi$ comes from the imposing that $n^u_\star$, which is dependent on $\phi$, be non-negative.
realization of $\phi = 1$, with appropriate accommodation in optimization problem and solution for these parameters. Note that in this version of the model, $\phi = S^{II}/\rho(n)S^{I}$.\(^{12}\)

Continue the analysis by making the VC’s budget constraint binding...

*Extension 1: introduce heterogeneity of projects.*

*Extension 2: get rid of representative VC and replace with a population of independent VCs. Analyze with simulations.*

\section*{II Conclusion}

To be developed

\footnote{As discussed in footnote (5), as solved, $n^*_v$ generates negative expected investment profits. This can be resolved by imposing the IC3 constraint in the VC’s optimization problem. In the presence of failing projects, IC3': $1 + k\rho(n) \leq F\rho(n)^2$.}
References


